

Algebra 2 Midterm Review KEY

(A) Simplifying

$$1) 4\sqrt{25} \div (9-4) \cdot 2$$

$$4 \cdot 5 \div 5 \cdot 2$$

$$20 \div 5 \cdot 2$$

$$4 \cdot 2$$

$$\textcircled{8}$$

$$2) 10 - \{22 \div [2 + (3 \cdot 3)]\}$$

$$10 - \{22 \div 11\}$$

$$10 - 2$$

$$\textcircled{8}$$

$$3) 5x^3 + 2x + 4x^2 - x$$
$$= \boxed{5x^3 + x + 4x^2}$$

$$4) (4n^2 - 6n + 3) - (\overbrace{7n + 3}^{\text{ } - 8n^2})$$
$$= 4n^2 - 6n + 3 - 7n - 3 + 8n^2$$
$$= \boxed{12n^2 - 13n}$$

$$5) 4(c + 2b) - 3(2c + b)$$
$$= 4c + 8b - 6c - 3b$$
$$= \boxed{-2c + 5b}$$

$$6) (x - 2y) - 4(\overbrace{-x + y}^{\text{ } }) + 2(\overbrace{-4x + 5y}^{\text{ } })$$
$$= x - 2y + 4x - 4y - 8x + 10y$$
$$= \boxed{-3x + 4y}$$

(B) Solving Equations

1) $2(x-3) + x = 4x + 2$

$$2x - 6 + x = 4x + 2$$

$$\begin{array}{r} 3x - 6 = 4x + 2 \\ -3x \quad -3x \end{array}$$

$$\begin{array}{r} -6 = x + 2 \\ -2 \quad -2 \end{array}$$

$$\boxed{-8 = x}$$

2) $5x + 6 = 36$

$$\begin{array}{r} -6 \quad -6 \end{array}$$

$$\begin{array}{r} 5x = 30 \\ \underline{5} \quad \underline{5} \end{array}$$

$$\boxed{x = 6}$$

3) $6x + 2 = 5x - 4$

$$\begin{array}{r} -5x \quad -5x \end{array}$$

$$\begin{array}{r} x + 2 = -4 \\ -2 \quad -2 \end{array}$$

$$\boxed{x = -6}$$

4) $v \cdot D = \frac{M}{x}$ ✓

$$\frac{v \cdot D}{D} = \frac{M}{D}$$

$$\boxed{v = \frac{M}{D}}$$

5) $2l + 2w = P$

$$\begin{array}{r} -2l \quad -2l \end{array}$$

$$\frac{2w}{2} = \frac{P - 2l}{2}$$

$$\boxed{w = \frac{P - 2l}{2}}$$

6) $3x^2 + 7 = y$

$$\begin{array}{r} -7 \quad -7 \end{array}$$

$$\frac{3x^2}{3} = \frac{y - 7}{3}$$

$$\sqrt{x^2} = \sqrt{\frac{y - 7}{3}}$$

$$\boxed{x = \sqrt{\frac{y - 7}{3}}}$$

(C) Radicals

$$1) 4\sqrt{40}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ \boxed{4} \quad 10 \end{array}$$

$$4\sqrt{4}\sqrt{10}$$

$$= \overset{1}{4} \cdot \overset{1}{2} \cdot \overset{1}{\sqrt{10}}$$

$$= \boxed{8\sqrt{10}}$$

$$2) \sqrt{68}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ \boxed{4} \quad 17 \end{array}$$

$$= \sqrt{4}\sqrt{17}$$

$$= \boxed{2\sqrt{17}}$$

$$3) \frac{10}{7\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{10\sqrt{3}}{7 \cdot 3} = \boxed{\frac{10\sqrt{3}}{21}}$$

$$4) \sqrt{112a^3b^2}$$

$$\begin{array}{c} \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ 16 \quad 7 \quad a^2 \quad a \end{array}$$

$$= \sqrt{16}\sqrt{7}\sqrt{a^2}\sqrt{a}\sqrt{b^2}$$


$$= \boxed{4ab\sqrt{7a}}$$


$$5) \sqrt[4]{8a^3b} = (8a^3b)^{\frac{1}{4}}$$


(D) Translating


$$1) n^3 - 10 \quad 2) 6(x+2) \quad 3) w = 5l - 3 \quad 4) (x-y)^3 = 8$$

(E) Function Families

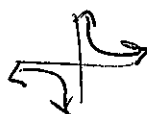
$$1) f(x) = x$$



$$2) f(x) = |x|$$


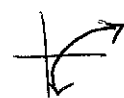
$$3) f(x) = x^2$$


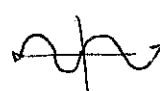
$$4) f(x) = x^3$$


$$5) f(x) = \sqrt{x}$$


$$6) f(x) = \frac{1}{x}$$


$$7) f(x) = e^x$$


$$8) f(x) = \ln x$$


$$9) f(x) = \sin x$$


(F) Testing Functions

1) No 2) yes 3) yes 4) no

(G) Domain + Range

1) $D: \mathbb{R}$

$R: [-3, \infty)$

2) $D: \{-4, 1, 2, 6\}$

$R: \{-1, 4, 8, 9\}$

3) $D: [-2, \infty)$

$R: \mathbb{R}$

(H) Evaluating + Combining

1) $(f+g)(x) = 6x - 3 + 2x^2 - 5x + 9$
 $= \boxed{2x^2 + x + 6}$

2) $(f-g)(x) = 6x - 3 - (2x^2 - 5x + 9)$
 $= 6x - 3 - 2x^2 + 5x - 9$
 $= \boxed{-2x^2 + 11x - 12}$

3) $(h \cdot g)(x) = 5x(2x^2 - 5x + 9)$
 $= \boxed{10x^3 - 25x^2 + 45x}$

4) $\left(\frac{g}{f}\right)(x) = \frac{2x^2 - 5x + 9}{6x - 3}$

5) $g(-3) = 2(-3)^2 - 5(-3) + 9$
 $= 2 \cdot 9 + 15 + 9$
 $= 18 + 15 + 9 = \boxed{42}$

(I) Composition

$$\begin{aligned} 1) h \circ f &= h[f(x)] = h(9x-4) \\ &= 3(9x-4) \\ &= \boxed{27x-12} \end{aligned}$$

$$\begin{aligned} 2) f \circ h &= f[h(x)] = f(3x) \\ &= 9(3x) - 4 \\ &= \boxed{27x-4} \end{aligned}$$

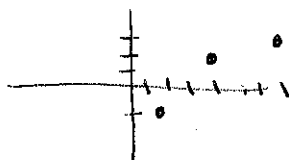
$$3) f[h(-4)]$$

$$h(-4) = 3(-4) = -12$$

$$f(-12) = 9(-12) - 4 = -108 - 4 = \boxed{-112}$$

(J) Inverse Functions

$$1) f^{-1} = \{(4, 2), (2, -1), (7, 3)\}$$



$$2) f(x) = 3x + 6$$

$$\begin{array}{r} y = 3x + 6 \\ -6 \quad -6 \end{array}$$

$$\frac{y-6}{3} = \frac{3x}{3}$$

$$\frac{y-6}{3} = x$$

$$\frac{1}{3}y - 2 = x$$

$$\frac{1}{3}x - 2 = y$$

$$\boxed{f^{-1} = \frac{1}{3}x - 2}$$

$$3) \frac{y+6}{5} = \frac{5x}{5}$$

$$\frac{1}{5}y + \frac{6}{5} = x$$

$$\frac{1}{5}x + \frac{6}{5} = y$$

$$\boxed{f^{-1} = \frac{1}{5}x + \frac{6}{5}}$$

(K) Writing Eq. of Line

1) $y = 5x + 7$ 2) $M = 3$ $(0, -\frac{1}{2}) \rightarrow$ means $b = -\frac{1}{2}$

$$y = 3x - \frac{1}{2}$$

3) $M = -2$ $(-4, 6)$
 x, y

$$y = -2x + b$$

$$b = -2(-4) + b$$

$$b = 8 + b$$

$$\begin{array}{r} -8 \quad -8 \\ \hline \end{array}$$

$$-2 = b$$

ae

x	y
-4	6
-3	4
-2	2
-1	0
0	-2

$$y = -2x - 2$$

4) $M = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13 - 7}{6 - 4} = \frac{6}{2} = 3$

$$y = 3x - 5$$

$$y = 3x + b$$

$$7 = 3(4) + b$$

$$7 = 12 + b$$

$$\begin{array}{r} -12 \quad -12 \\ \hline -5 = b \end{array}$$

Now plug in
either point
ex: $(4, 7)$
 x, y

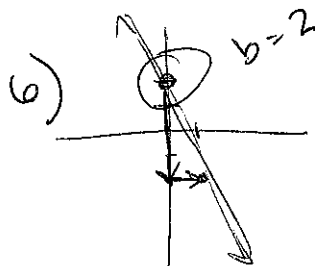
5)

x	y
-1	4
0	6
1	8

 $+1 < -1 > +2$
 $+1 < 1 > +2 \rightarrow b = 6$

$$M = \frac{2}{1} = 2$$

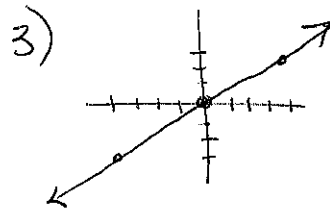
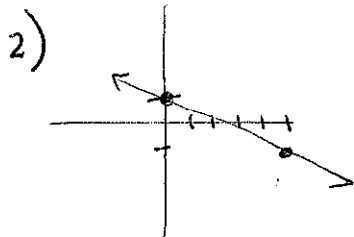
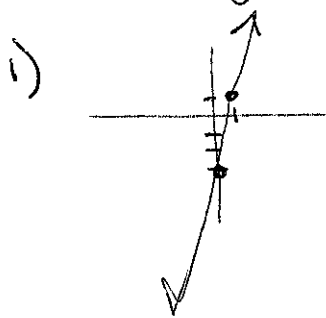
$$y = 2x + 6$$



$$M = \frac{\Delta y}{\Delta x} = \frac{-4}{1} = -4$$

$$y = -4x + 2$$

(L) Graphing



(M) Transformations

1) $y = -4x + 2$

↙ reflected ↘ shift up 2

steeper

2) $y = \frac{1}{3}x - 4$

↙ flatter ↘ shift down 4

(N) Linear Applications

1) cost for each = $3.50 + 0.75 = \$4.25$ per banner

Profit = in - out

$= 11 - 4.25 = \$6.75$ per banner

$\frac{500}{6.75} = 74.07 \Rightarrow$ need to sell 75 banners

2) a. $m = 12$ - cost per box b. $b = 100$ - flat rate

c. independent = # boxes
dependent = cost

d. $y = 12x + 100$

e. $f(15) = 12(15) + 100$
 $= \$280$

f.

D	R
# boxes	\$
1	112
2	124
3	136
4	148
...	...
15	280

D: $\{1, 2, 3, 4, \dots, 15\}$

R: $\{112, 124, 136, 148, \dots, 280\}$

① Systems & Equations

1) C = chickens P = pigs

animals : $C + P = 13$

legs : $2C + 4P = 40$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 1 & 13 \\ 2 & 4 & 40 \end{array} \right] \xrightarrow{\text{ref}} \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 7 \end{array} \right] \begin{matrix} \rightarrow C = 6 \\ \rightarrow P = 7 \end{matrix}$$

6 chicken + 7 pigs

2) F = fancy
P = plain

$$F + P = 7$$

$$28F + 15P = 131$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 1 & 7 \\ 28 & 15 & 131 \end{array} \right] \xrightarrow{\text{ref}} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 5 \end{array} \right] \begin{matrix} \rightarrow F = 2 \\ \rightarrow P = 5 \end{matrix}$$

2 fancy & 5 plain

3) d = hot dogs

h = hamburgers

s = soft drinks = ?

$$d = 2h$$

$$s = 3h$$

$$1d + h + s = 720$$

Method 1: $d + h + s = 720$

$$2h + h + 3h = 720$$

$$6h = 720$$

$$h = 120$$

$$s = 3h$$

$$= 3(120)$$

$$= 360 \text{ soft drinks}$$

Method 2: $d = 2h$
 $-2h \quad -2h$

$$d - 2h = 0$$

$$s = 3h$$

$$-3h \quad -3h$$

$$s - 3h = 0$$

$$d + h + s = 720$$

$$d - 2h = 0$$

$$-3h + s = 0$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 720 \\ 1 & -2 & 0 & 0 \\ 0 & -3 & 1 & 0 \end{array} \right]$$

$$\xrightarrow{\text{ref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 240 \\ 0 & 1 & 0 & 120 \\ 0 & 0 & 1 & 360 \end{array} \right]$$

360 soft drinks